Astrophys Space Sci
DOI 10.1007/s10509-014-2222-2

ERRATUM

Erratum to: Benchmarking the unified nonlinear transport theory for Goldreich–Sridhar turbulence

A. Shalchi · M. Hussein

Received: 9 December 2014 / Accepted: 9 December 2014

Erratum to: Astrophys Space Sci (2013) 344:187–191 DOI 10.1007/s10509-012-1298-9

In Shalchi (2013) the test-particle simulations performed by Sun and Jokipii (2011) were compared with the so-called Unified Non-Linear Transport (UNLT) theory developed in Shalchi (2010). These numerical and analytical results were obtained by employing a turbulence model based on Goldreich and Sridhar (1995) scaling. It was shown that the UNLT theory agrees very well with the simulations. Therefore, it was concluded that UNLT theory is an accurate analytical theory for particle transport across the mean magnetic field.

Sun and Jokipii (2011) listed parallel and perpendicular diffusion coefficients whereas the UNLT theory contains the corresponding mean free paths. These two parameters are related to each other via $\lambda_\parallel = 3\kappa_\parallel/v$ and $\lambda_\perp = 3\kappa_\perp/v$, respectively. The mean free paths $\lambda_\parallel$ and $\lambda_\perp$ have length units and, therefore, these parameters are usually normalized with respect the a characteristic scale of the turbulence L (see Shalchi 2013 for details). The parameter v denotes the particle speed. In Shalchi (2013) the approximation $v \approx c$ (where c is the speed of light) was employed to convert the simulations to mean free paths corresponding to the assumption that all particles move with relativistic speeds. This assumption was incorrect and, thus, we provide the correct table and a revised figure in this erratum.

The online version of the original article can be found under doi:10.1007/s10509-012-1298-9.

A. Shalchi (✉) · M. Hussein
Department of Physics and Astronomy, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada
e-mail: andreasm4@yahoo.com

M. Hussein
e-mail: m_hussein@physics.umanitoba.ca

The kinetic energy E_{kin} for a relativistic particle is given by:

$$E_{kin} = (\gamma - 1) m_0 c^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) m_0 c^2 \qquad (1)$$

where we have used the Lorentz factor γ and the rest mass of the particle m_0. The latter formula can be re-arranged to obtain the particle velocity as a function of kinetic energy

$$\frac{v}{c} = \sqrt{1 - \left(\frac{E_{kin}}{m_0 c^2} + 1 \right)^{-2}}. \qquad (2)$$

Sun and Jokipii (2011) considered protons and, therefore, the rest mass is given by $m_0 c^2 \simeq 938$ MeV. As indicated by Table 1, most of the considered particle energies were below the rest energy and, thus, the corresponding particles propagated non-relativistically.

The mean free paths normalized to the characteristic scale L are related to diffusion coefficients via

$$\frac{\lambda_i}{L} = \frac{3}{vL} \kappa_i \qquad (3)$$

where $i \in \{\parallel, \perp\}$. Thus, one has to know the product vL in order to convert diffusion coefficients into mean free paths normalized to the scale L. Sun and Jokipii (2011) used for the characteristic scale $L = 0.01$ AU.[1] In Shalchi (2013) the approximation $vL \approx cL = 45 \times 10^{20}$ cm^2/s was used which is only correct for relativistic protons. By using Eq. (2) and the values of the kinetic energy listed in Table 1, one can easily calculate the correct values of vL. The results are summarized in Table 1.

[1]Here we have used *Astronomical Units* which are related to metric units via 1 AU $\simeq 1.496 \times 10^{13}$ cm $\equiv 1.496 \times 10^{11}$ m.

Published online: 20 December 2014

Table 1 Kinetic energies and the correct products vL obtained from Sun and Jokipii (2011). Here we have used $cL = 45 \times 10^{20}$ cm^2/s

Run	E_{kin} in MeV	v/c	vL in 10^{20} cm^2/s
1	1.0	0.046	2.08
2	3.16	0.082	3.68
3	10.0	0.14	6.52
4	31.6	0.25	11.39
5	100	0.43	19.27
6	316	0.66	29.87
7	1000	0.88	39.38

Table 2 The correct converted parameters from Sun and Jokipii (2011). One can easily see that the ratio $\lambda_\perp/\lambda_\parallel$ is the same as in Shalchi (2013)

Run	R_L/L	$(\delta B/B_0)^2$	$\lambda_\parallel/L$	$\lambda_\perp/L$	$\lambda_\perp/\lambda_\parallel$
1	0.019	1.0	1.5111	0.0467	0.0309
2	0.034	1.0	1.7961	0.0525	0.02923
3	0.061	1.0	2.2107	0.0626	0.02832
4	0.109	1.0	3.2971	0.0714	0.021655
5	0.198	1.0	4.2487	0.0835	0.01965
6	0.37	1.0	6.7682	0.1191	0.01759
7	0.759	1.0	12.4819	0.1790	0.01434

Especially the first values of vL are much smaller than the $cL = 45 \times 10^{20}$ cm^2/s used in Shalchi (2013). Therefore, we obtain larger parallel and perpendicular mean free paths if the correct conversion is employed. The correct values of $\lambda_\parallel/L$ and $\lambda_\perp/L$ are listed in Table 2 and in Fig. 1 we show the corrected ratio $\lambda_\perp/L$ versus $\lambda_\parallel/L$.

One can easily see from Fig. 1 that the agreement between UNLT theory and the simulations is still remarkable. The only difference between Fig. 1 of Shalchi (2013) and the new figure is that the simulations are now closer to the analytical results obtained for $a^2 = 1$ rather then $a^2 = 1/3$. Therefore, the conclusions of Shalchi (2013) are still correct. UNLT theory agrees very well with test-particle simulations performed for Goldreich–Sridhar turbulence.

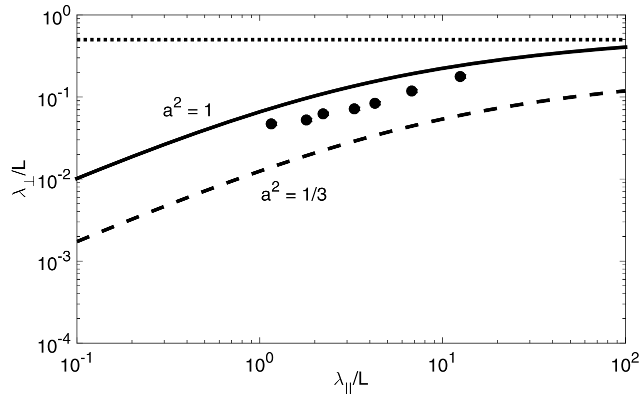

Fig. 1 The corrected perpendicular mean free path $\lambda_\perp$ versus the parallel mean free path $\lambda_\parallel$. Both parameters are normalized with respect to the characteristic turbulence scale L. Shown are the mean free paths obtained from the simulations performed by Sun and Jokipii (2011) for Goldreich–Sridhar turbulence (*dots*). Also shown are the mean free paths obtained from the *UNLT theory* for two different values of a^2, namely $a^2 = 1/3$ (*dashed line*) and $a^2 = 1$ (*solid line*). Also the quasi-linear perpendicular mean free path (see Eq. (3) from Sun and Jokipii 2011) is shown (*dotted line*)

Acknowledgements Andreas Shalchi acknowledges support by the Natural Sciences and Engineering Research Council (NSERC) of Canada.

References

Goldreich, P., Sridhar, S.: Astrophys. J. **438**, 763 (1995)
Shalchi, A.: Astrophys. J. **720**, L127 (2010)
Shalchi, A.: Astrophys. Space Sci. **344**, 187 (2013)
Sun, P., Jokipii, J.R.: In: 32nd International Cosmic Ray Conference, Beijing (2011)